

Quantum information transport to multiple receivers.

Andrew D. Greentree, Simon J. Devitt, and Lloyd C. L. Hollenberg
*Centre for Quantum Computer Technology, School of Physics,
 The University of Melbourne, Melbourne, Victoria 3010, Australia.*
 (Dated: February 1, 2008)

The importance of transporting quantum information and entanglement with high fidelity cannot be overemphasized. We present a scheme based on adiabatic passage that allows for transportation of a qubit, operator measurements and entanglement, using a 1-D array of quantum sites with a single sender (Alice) and multiple receivers (Bobs). Alice need not know which Bob is the receiver, and if several Bobs try to receive the signal, they obtain a superposition state which can be used to realize two-qubit operator measurements for the generation of maximally entangled states.

PACS numbers: 03.67.Hk, 72.25.Dc, 73.21.La

Allied to the efforts to build a working quantum computer (QC) is the requirement to replicate, in a quantum framework, the necessary features of a classical computer. In particular, some kind of quantum bus would be highly advantageous to allow a form of distributed QC [1]. Long range quantum information transfer usually exploits teleportation [2], or flying qubits [3]; we consider a mechanism more related to a quantum wire or fanout. Fanout operations are forbidden quantum mechanically, as they necessarily imply cloning, however, considering the closest quantum analog leads to a new approach to quantum communication, described here.

A naïve approach to transport in quantum system is via sequential swap gates between sites. This approach is often undesirable due to, for example, noise introduced by sensitive nonadiabatic controls, poor level of gate control, insufficient bandwidth or impractical gate density [2]. Many authors have begun to examine alternatives [4, 5, 6] considering schemes where a desired coupling is set up (usually statically) and the system allowed to evolve until the information transfer has occurred. In such schemes the receivers may be passive [5], or active [6], but it is usually assumed that neither sender nor receiver can modify the qubit chain, except for control of their own qubit or qubits and local coupling to the chain.

We propose an extremely general alternative for adiabatic transfer of a particle between positional quantum states. An obvious application of this is as a transport mechanism for ion trap QCs. In one approach [7] a scheme for transporting ions sequentially from storage zones to interaction sites was proposed via a microtrap array [8]: the Quantum Charge Coupled Device. With minor modification, our scheme provides an interesting alternative. One attractive feature of the present scheme is that requirements on quantum state guidance are minimized, and sympathetic cooling following transport should not be required. One could also consider solid-state realizations of this scheme in a patterned GaAs quantum dot array [9, 10] or where the confining potentials are realized using ionized P donors in a Si matrix [11, 12, 13].

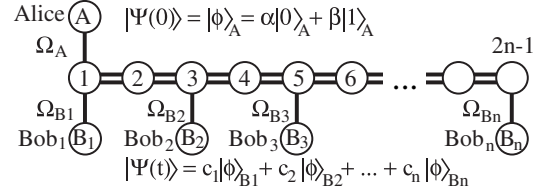


FIG. 1: Quantum bus for qubit transfer from Alice to the Bobs. The initial state $|\Psi(0)\rangle = |\phi\rangle_A$ goes to a spatial superposition state $|\Psi(t)\rangle = \sum_j c_j |\phi\rangle_j$. The bus is defined by tunnel coupled sites 1 to n . Static, high tunneling matrix elements (TME) are shown by double lines, lower, controlled TME by single lines. Alice controls one site, A , and the Bobs are represented by the sites B_i , connected to the bus at sites $1, 3 \dots 2n-1$. Alice controls Ω_A , and Bob j controls Ω_{B_j} .

We consider a quasi-one-dimensional chain of quantum sites, realized, by the empty or singly occupied states of a positional eigenstate, see Fig. 1. The sender (Alice) distributes information via a qubit using the chain to a series of receivers (Bob₁ to Bob_n). Alice and each Bob control one site each, attached to the central chain, and Alice need not know who receives the signal.

Our scheme differs from previously discussed schemes by the use of spatial adiabatic passage to transport the qubit. The addition of multiple receivers and its inherent flexibility is a significant departure from previous adiabatic protocols [14, 15, 16, 17], and we term this new transport scheme MRAP - Multiple Receiver Adiabatic Passage. In addition, the previous adiabatic passage protocols are further augmented by transporting an extra, spectator degree of freedom in addition to the spatial degree of freedom. This allows flexible information transfer, and even distributed entanglement in an adiabatic context. The geometry is chosen to ensure that there is always an odd number of quantum sites between Alice and each Bob, see Fig. 1, which is required by the adiabatic passage protocol [15], analogous versions of the three-site protocol have been considered in optical lattices [16] and Cooper-Pair boxes [17], and a two-electron, three-site variant has been suggested for entanglement distilla-

tion [18]. Although we explicitly consider spatial transfer of particles, information could be transferred using spin chains (e.g. that of Ref. 5) with time-varying adiabatic coupling sequences instead of static couplings.

To investigate transport, we write the Hamiltonian for a single qubit carried by a particle in a positional array

$$\mathcal{H} = \sum_{\sigma=0,1} \left[\sum_{i=1}^{2n-1} \left(\frac{E_{i,\sigma}}{2} c_{i,\sigma}^\dagger c_{i,\sigma} + \Omega_S c_{i+1,\sigma}^\dagger c_{i,\sigma} \right) + \left(\frac{E_A}{2} c_{A,\sigma}^\dagger c_{A,\sigma} + \Omega_A c_{1,\sigma}^\dagger c_{A,\sigma} \right) + \sum_{j=1}^n \left(\frac{E_{B_j}}{2} c_{B_j,\sigma}^\dagger c_{B_j,\sigma} + \Omega_{B_j} c_{B_j,\sigma}^\dagger c_{2j+1,\sigma} \right) \right] + \text{h.c.}, \quad (1)$$

where we have introduced the (externally controlled) site energies, E , and Ω_S is the tunneling matrix element (TME) along the bus, which is not varied during the protocol, Ω_A is the TME between A and 1 which Alice can control, whilst Ω_{B_j} is the TME between B_j and $2j-1$, and $c_{i,\sigma}$ is the annihilation operator for a qubit with state $\sigma = 0, 1$ on site i for $i = A, 1 \dots 2n-1, B_1 \dots B_n$. Control of these TMEs is by varying the potential barrier between the sites and the chain. The exact method for this variation is implementation dependent, but for a GaAs or P:Si system could be via surface gates [10, 12], or mean well separation in an optical lattice [16]. For notational brevity we do not indicate unoccupied sites explicitly, so that $c_{i,\sigma}^\dagger |\text{vac}\rangle = |\sigma\rangle_i$. Eq. 1 comprises three terms, the first corresponds to the energy of the particle in the sites on the chain, and the TMEs between chain sites, the second to the energy of the particle on Alice's site, and the coupling from Alice to the chain, and the final term to the energy of the particle at the Bobs' sites and their tunneling to the chain. For n Bobs there must be at least $2n-1$ sites in the chain, so we assume this number (extra sites in the chain do not interfere with the scheme as discussed below). As the qubit degree of freedom is decoupled from the positional degree of freedom, it is carried along as a 'spectator' storing information, but otherwise unaffected by the transfer.

To realize the counter-intuitive pulse sequence, we set all of the site energies to 0 (i.e. $E_{i,\sigma} = 0$ for $i = 1 \dots 2n-1, A, B_1 \dots B_n$), using the external control. The TMEs are modulated (again via external control) in a Gaussian fashion according to (see Fig. 2)

$$\Omega_A(t) = \Omega^{\max} \exp \left\{ -([t - (t_{\max}/2 + s)]^2 / (2s^2)) \right\}, \\ \Omega_{B_j}(t) = \Omega_{B_j}^{\max} \exp \left\{ -([t - (t_{\max}/2 - s)]^2 / (2s^2)) \right\}, \quad (2)$$

where $\Omega^{\max} \ll \Omega_S$, and s is the width of the applied pulses. $\Omega_{B_j}^{\max} = \Omega^{\max}$ if Bob $_j$ wishes to receive a signal from Alice, and $\Omega_{B_j}^{\max} = 0$ otherwise. The scheme is extremely robust to the choice of modulation, and in common with conventional adiabatic transfer schemes alternatives to Gaussians have little effect providing the

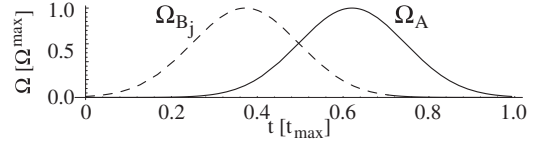


FIG. 2: Counter-intuitive sequence for transport from Alice to Bobs. Each receiver Bob chooses the same coherent tunneling profile. On this scale, the TME between bus sites would be at least 10. Increasing the length of time for the protocol increases the fidelity, up to the limits allowed by dephasing.

adiabaticity criterion is satisfied [14]. As no control of the chain is possible apart from modifications of the TMEs in the vicinity of Alice and Bob, Ω_S is constant.

The first case to consider is a chain where only the final Bob chooses to receive the qubit from Alice, i.e. $\Omega_{B_i} = 0, i = 1 \dots n-1$. In this case the MRAP protocol reduces to previous adiabatic protocols [15] and will not be treated here. More generally, however, we show that the protocol works for *any* Bob, or indeed for multiple Bobs *simultaneously*: quantum fanout.

Alice broadcasts her qubit, and the Bobs have colluded so there is only one receiver, Bob $_j$, which is not communicated to Alice. Remarkably, no extension to the above scheme is required. When both Alice and Bob $_j$ perform MRAP, Bob $_j$ receives the information in a time-scale defined by the *total* length of the bus, and *independent* of the site where Bob $_j$ is situated. In marked departure from most previous schemes, the information does not propagate along the bus. Rather, one eigenstate is smoothly transformed from being located at Alice's site, to Bob $_j$'s site, without occupying the bus at any stage. To understand this, consider the two-Bob protocol (with three chain sites), with Alice and Bob $_1$ connected to site 1, and Bob $_2$ connected to site 3. Adiabatic transport utilizes the eigenstates of Eq. 1 which have zero eigenvalue, i.e. the two-dimensional null space of \mathcal{H} , spanned by

$$|\psi_1\rangle = (\Omega_{B_1}|\phi\rangle_A - \Omega_A|\phi\rangle_{B_1}) \sqrt{\Omega_A^2 + \Omega_{B_1}^2}, \\ |\psi_2\rangle = \frac{\Omega_{B_2}|\phi\rangle_A - \frac{\Omega_A\Omega_{B_2}}{\Omega_S}|\phi\rangle_2 + \Omega_A|\phi\rangle_{B_2}}{\sqrt{\Omega_A^2 + \Omega_{B_2}^2}}, \quad (3)$$

where $\Omega_S \gg \Omega_A, \Omega_{B_1}, \Omega_{B_2}$, and $|\phi\rangle_i \equiv \alpha|0\rangle_i + \beta|1\rangle_i$. Any linear combination of these two states is also a null state, so it suffices to find the state adiabatically connected to Alice's site at $t = 0$, i.e. $|\Psi(t=0)\rangle = |\phi\rangle_A$. If $\Omega_{B_2}(t) = 0 \forall t$, $|\psi_1\rangle$ is adiabatically connected to $|\Psi(t=0)\rangle$ and the qubit is transferred from $|\phi\rangle_A$ to $|\phi\rangle_{B_1}$, if $\Omega_{B_1}(t) = 0 \forall t$, then $|\psi_2\rangle$ is adiabatically connected to $|\phi\rangle_A$, and the qubit transferred from $|\phi\rangle_A$ to $|\phi\rangle_{B_2}$. Hence the qubit can be sent from Alice to either Bob, without Alice knowing which Bob is the receiver.

If both Bobs are receivers, they choose $\Omega_{B_1}(t) = \Omega_{B_2}(t)$ and $(|\psi_1\rangle - |\psi_2\rangle)/2$ is adiabatically connected

to $|\phi\rangle_A$. The final state of the system after MRAP is $(|\phi\rangle_{B_1} - |\phi\rangle_{B_2})/\sqrt{2}$, which is quantum fanout, with both Bobs sharing an equal positional superposition of the qubit. We stress that have not cloned Alice's qubit, and measurements of the qubit position will collapse it at either B_1 or B_2 . For adiabatic transport, the adiabaticity criterion [14] must be satisfied, i.e., the inverse transfer time must be small compared with the energy gap between states, $E = [(\Omega_A^2 + \Omega_{B_1}^2 + \Omega_{B_2}^2)/2]^{1/2}$ for large Ω_S .

In the general case the null space is spanned by

$$|\psi_j\rangle = \frac{\Omega_{B_j}|\phi\rangle_A + \sum_{k=1}^{j-1} \frac{\Omega_A \Omega_{B_j}}{(-1)^k \Omega_S} |\phi\rangle_{2k} + (-1)^j \Omega_A |\phi\rangle_{B_j}}{\sqrt{\Omega_A^2 + \Omega_{B_j}^2}}, \quad (4)$$

and up to known signs, all receiver Bobs obtain an equal superposition of the qubit. One can show that the energy gap between the zero and next nearest eigenstate is $E_{\text{gap}} = [(\Omega_A^2 + \sum_{k=1}^j \Omega_{B_k}^2)/j]^{1/2}$, for the j -Bob protocol, so the scaling for more Bobs goes as $[(1+j)/j]^{1/2}$.

We have performed preliminary studies of the robustness of this scheme, and find a similar resistance to errors as other adiabatic protocols. If the simultaneity of the Bob pulses is not exact, or if the Ω_B pulse areas are not exactly the same, then the superposition state shared following the protocol will not be exact. Providing adiabaticity is satisfied, such errors introduce a monotonic decrease in fidelity. It is difficult to explore the full state space because of the large range of parameters, however, the point is that MRAP is not exponentially sensitive to errors. We have also solved the case where the chain is cyclic, i.e. where site 1 is connected to sites 2 and $2n$, and the protocol works without modification. These results will be presented in more detail elsewhere.

We have shown the most general case of Alice sending a qubit to the Bobs, but a special case would be where Alice could be a factory of pure states. MRAP could be used to send these states with high fidelity around a quantum network, important for many QC architectures.

The only difference between Alice and Bobs is the order in which they vary their coupling to the bus. Therefore the Bobs can also perform inter-Bob communication by assuming the role of Alice or Bob as required. Reversing the protocol for the two-Bob case (with $\Omega_{B_1} = \Omega_{B_2}$) gives the transformations

$$|\phi\rangle_{B_1} \Rightarrow (1/\sqrt{2})|\phi\rangle_A + (1/2)(|\phi\rangle_{B_1} + |\phi\rangle_{B_2}), \quad (5)$$

$$|\phi\rangle_{B_2} \Rightarrow -(1/\sqrt{2})|\phi\rangle_A + (1/2)(|\phi\rangle_{B_1} + |\phi\rangle_{B_2}). \quad (6)$$

We have not included the effects of dephasing here, as it will vary significantly between different implementations. In most practical systems of particle transfer, one expects particle localization to dominate over spontaneous emission, i.e. $T_2 \ll T_1$. Hence we will ignore T_1 . However, T_2 processes will take the system out of the null



FIG. 3: Each Bob has, in addition to their site, B_j , a qubit, Q_j , and a CNOT or CZ gate with B_j as control and Q_j as target. This allows MRAP to be used for operator measurements and entanglement between the Q_j .

space, and will be detrimental to the transfer protocol. Dephasing has been considered analytically in STIRAP [21] and numerically for higher order protocols [13]. If the minimum transfer time is satisfied, the transfer failure probability goes as $\Gamma_2 T_{\text{tot}}$ where Γ_2 is the dephasing rate and T_{tot} is the *total* protocol time, and this requirement is no different from the requirements for charge-qubit systems. Our numerical models show that MRAP is not inherently more sensitive than the 1-D spatial adiabatic passage, although the total time that superposition states must be maintained will be longer than in the 1-D case, with corresponding (linear) decrease of robustness.

Considering implementations, the total time for the protocol must be $T_{\text{tot}} \gtrsim 10/\Omega_{\text{max}}$ [15], and the chain TMEs must be $\Omega_S \gtrsim 10\Omega_{\text{max}}$. In a P:Si system [13], with Alice and Bob site separations from the chain of 30nm, and interchain separations of 20nm, give rise to $\Omega_{\text{max}} \sim 100\text{GHz}$ and $\Omega_S \sim 1\text{THz}$, which gives a probability of transfer error of 10^{-2} for $\Gamma_2 = 100\text{MHz}$ for $T_{\text{tot}} \sim 2\text{ns}$, which is certainly feasible (though unmeasured) given current projections of P:Si architectures. Hu *et al.* [22] suggest that GaAs quantum dots with TMEs in the same range could be achieved with inter-dot spacings of 30 – 35nm. Petta *et al.* [23] measured charge dephasing rates of $\Gamma_2 \sim 10\text{GHz}$, suggesting that a proof of principle demonstration of MRAP is already possible, although improvements in Γ_2 are needed before a practical GaAs implementation is possible. Ion trap and optical lattice systems, however, show the most promise for demonstrations. Eckert *et al.* [16] estimate adiabatic timescales for the three-state protocol of order milliseconds, and because the transfer is in vacuum, Γ_2 should be small compared with this rate.

MRAP can be extended to realize two-qubit operator measurements [19]. The extension requires augmenting the existing protocol with extra qubits and the ability to perform entanglement operations between the Bob sites and these qubits. By using the bus as a mediator for entanglement, our protocol has a similar aim to the multisplitter of Paternostro *et al.* [20], but arises from a very different mechanism. In addition to their vacant sites, B_j , each Bob has a qubit, Q_j , and can perform either a CNOT or CZ operation between sites B_j (control) and Q_j (target), depicted in Fig. 3. An example, we show the protocol for two-Bob MRAP, where the multi-Bob bus forms an effective qutrit ancilla (formed by the states $|1\rangle_A, |1\rangle_{B_1}, |1\rangle_{B_2}$), and we demonstrate projective

measurements of $U_1 U_2 \equiv \sigma_{U, Q_1} \otimes \sigma_{U, Q_2}$ for $U = X, Z$.

1. Initially, the Q_j 's are in the arbitrary state $|\Phi\rangle_{Q_1, Q_2}$, and the bus in state $|1\rangle_A$. The total system state is

$$|\Psi\rangle = |1\rangle_A |\Phi\rangle_{Q_1, Q_2}.$$

2. MRAP is performed, the system's state becomes

$$|\Psi\rangle = (1/\sqrt{2}) (|1\rangle_{B_1} - |1\rangle_{B_2}) |\Phi\rangle_{Q_1, Q_2}.$$

3. The Bobs perform a Controlled- U operation between sites B_i and Q_i , where the action of the controlled operation is trivial when B_i is unoccupied, the state evolves to

$$|\Psi\rangle = (1/\sqrt{2}) (|1\rangle_{B_1} U_{Q_1} I_{Q_2} - |1\rangle_{B_2} I_{Q_1} U_{Q_2}) |\Phi\rangle_{Q_1, Q_2},$$

where I is the identity operator.

4. MRAP transfer is reversed, generating the state

$$|\Psi\rangle = (1/2) |1\rangle_A (U_{Q_1} I_{Q_2} + I_{Q_1} U_{Q_2}) |\Phi\rangle_{Q_1, Q_2} + (2\sqrt{2})^{-1} (|1\rangle_{B_1} + |1\rangle_{B_2}) (U_{Q_1} I_{Q_2} - I_{Q_1} U_{Q_2}) |\Phi\rangle_{Q_1, Q_2}.$$

5. A measurement is performed at Alice, detecting the bus qubit with probability $1/4$, and projecting the state of $[Q_1, Q_2]$ to $(U_{Q_1} I_{Q_2} + I_{Q_1} U_{Q_2}) |\Phi\rangle_{Q_1, Q_2}$, i.e. the $+1$ eigenstate of $U_{Q_1} U_{Q_2}$, if successful.

6. If no qubit was measured at Alice, the system is projected to $(U_{Q_1} I_{Q_2} - I_{Q_1} U_{Q_2}) |\Phi\rangle_{Q_1, Q_2}$, which is the -1 eigenstate of $U_{Q_1} U_{Q_2}$, so a σ_z at B_2 allows the qubit to be deterministically returned to Alice in another reverse of the MRAP protocol. Hence MRAP affords a complete two-qubit operator measurement of XX and ZZ .

As the above protocol gives XX and ZZ operator measurements on physically separated qubits, we may use this to create multi-particle stabilizer states, e.g. the N particle GHZ state $|GHZ\rangle_N = (1/2)^{N/2} (|00\dots 0\rangle_N + |11\dots 1\rangle_N)$ (for convenience we choose N even). First, initialize Q_1 to Q_N to $|0\rangle$. Then, perform $X_{2i-1} X_{2i}$ stabilizer measurements via two-Bob MRAP on the pairs of Q_{2i-1} and Q_{2i} for $i = 1\dots N/2$, creating a series of independent two-particle Bell states $\bigotimes_i (|10\rangle \pm |01\rangle)_{Q_{2i-1} Q_{2i}}$ where the relative sign is known from the projective measurement result at Alice. Local single qubit operations can be used to convert the Bell pairs to $|00\rangle + |11\rangle$. Next $Z_{2i} Z_{2i+1}$ stabilizer measurements are performed between the Bell pairs, and this is sufficient to project the computer into $|GHZ\rangle_N$ (up to local operations). If one has access to multiple Alice's, and the ability to break the chain freely (with surface gates), then these operations can be performed in parallel, meaning that the $|GHZ\rangle_N$ state can be formed in two MRAP steps.

We have introduced a transport mechanism for quantum information around a network, based on adiabatic passage. With minor modification our scheme can also be used for generating entanglement and two-qubit measurements. The scheme is ideally suited as an alternative

to conventional ionic transport in an ion trap quantum computer. However its utility not restricted to ion traps, and it should have wide applicability to all architectures, especially solid-state quantum computing architectures.

ADG thanks Fujitsu for support whilst at University of Cambridge, and discussions with S. G. Schirmer, D. K. L. Oi, J. H. Cole, A. G. Fowler and C. J. Wellard. LCLH was supported by the Alexander von Humboldt Foundation, and thanks the van Delft group at LMU for their hospitality, and discussions with F. Wilhelm. This work was supported by the Australian Research Council, US National Security Agency (NSA), Advanced Research and Development Activity (ARDA) and Army Research Office (ARO) under contract W911NF-04-1-0290.

-
- [1] J. Eisert *et al.*, Phys. Rev. A **62** 052317 (2000).
 - [2] D. Copsey *et al.*, IEEE Journal of Selected Topics In Quantum Electronics **9**, 1552 (2003).
 - [3] D. P. DiVincenzo, Fort. der Physik **48**, 771 (2000).
 - [4] M. B. Plenio, J. Hartley, and J. Eisert, New J. of Phys. **6**, 36 (2004); M. H. Yung and S. Bose, Phys. Rev. A **71**, 032310 (2005); M. Paternostro *et al.*, *ibid.*, 042311 (2005); D. Burgarth and S. Bose, *ibid.* 052315 (2005).
 - [5] M. Christandl *et al.*, Phys. Rev. Lett. **92**, 187902 (2004).
 - [6] H. L. Haselgrove, quant-ph/0404152.
 - [7] D. Kielpinski, C. Monroe, D. J. Wineland, Nature (London) **417**, 709 (2002).
 - [8] J. I. Cirac and P. Zoller, Nature (London) **404**, 579 (2000).
 - [9] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); J. M. Elzerman *et al.*, Nature (London) **430**, 431-435 (2004); A. C. Johnson *et al.*, cond-mat/0410679.
 - [10] T. Hayashi *et al.* Phys. Rev. Lett. **91**, 226804 (2003).
 - [11] B. E. Kane Nature **393**, 133 (1998); R. G. Clark *et al.* Philos. Trans. R. Soc. London, Ser. A **361**, 1451 (2003); T. Schenkel *et al.*, J. Appl. Phys. **94**, 7017 (2003);
 - [12] L. C. L. Hollenberg *et al.* Phys. Rev. B **69**, 113301 (2004).
 - [13] L. C. L. Hollenberg *et al.*, quant-ph/0506198.
 - [14] N. V. Vitanov *et al.*, Annu. Rev. Phys. Chem. **52**, 763 (2001).
 - [15] A. D. Greentree *et al.*, Phys. Rev. B **70**, 235317 (2004).
 - [16] K. Eckert *et al.*, Phys. Rev. A **70**, 023606 (2004).
 - [17] J. Siewert and T. Brandes, Adv. in Solid State Phys. **44**, 181 (2004).
 - [18] J. Fabian and U. Hohenester, cond-mat/0412229.
 - [19] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 - [20] M. Paternostro, H. McAneney, and M. S. Kim, Phys. Rev. Lett **94**, 070501 (2005).
 - [21] Q. Shi and E. Gevba, J. Chem. Phys. **119**, 11773 (2003); P. A. Ivanov, N. V. Vitanov, and K. Bergmann, Phys. Rev. A **70**, 063409 (2004).
 - [22] X. Hu, B. Koiller, and S. Das Sarma, Phys. Rev. B **71**, 235332 (2005)
 - [23] J. R. Petta *et al.* Phys. Rev. Lett. **93**, 186802 (2004).